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Suppression Method for Torque Ripple of PM Synchronous Motor

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ABSTRACT

A new modified trapezoidal modulating signal for a pulse width modulation (PWM) inverter suitable for a permanent magnet synchronous motor (PMSM) drive is proposed in this paper. A new modulating signal for the PMSM drive is determined by the characteristic torque ripple of the motor with various electro-motive force (EMF). The proposed modulating signal is able to decrease the torque ripple even if the motor has sinusoidal EMF or non-sinusoidal EMF. By using the proposed modulating signal, the system reduced the torque ripple as well as achieved the effective utilization of the DC supply voltage for the inverter. Many improvements are accomplished by the PWM strategy adapting the modified trapezoidal modulating signal without a change in hardware.

Keywords: PMSM, Torque ripple, Electro-motive force, Modified trapezoidal modulating signal

1. Introduction

Increasing the amplitude of the fundamental component in the output waveform of the inverter achieves an effective utilization of the DC power supply, reduces the rated capacity of the inverter elements and decreases the turn-on losses of the switching devices. This is accomplished by the PWM strategy without a change in hardware. In general, a change in software does not involve extra complexity or increase in price. Therefore, the improvements by mean of the PWM strategy are an important and significant matter. The output waveform in the inverter increases the amplitude of the fundamental components as non-sinusoidal waves. AC motors driven by the non-sinusoidal PWM strategy generates the torque ripple. Therefore, the inverter for the motor drive generally improves the output voltage waveform by removing the low-order harmonics.

We have proposed a PWM strategy using the modified trapezoidal modulating signal suitable for induction motor drives [1]. A modified trapezoidal modulating signal for the induction motor drive is determined by reducing the torque ripple of the motor. By using the proposed modulating signal, the system reduced the torque ripple as well as achieved the effective utilization of the DC supply voltage for the inverter. Many improvements are accomplished by the PWM strategy while adapting the modified trapezoidal modulating signal without a change in hardware. Moreover, a pulse amplitude modulation (PAM) inverter using modified trapezoidal modulating signal has been proposed [2,3]. The PAM inverter for

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induction motor drive has several advantages [2,4].

In this paper, a new modified trapezoidal modulating signal suitable for PMSM drive is proposed. A new modified trapezoidal modulating signal for the PMSM drive is determined by characteristic torque ripple of the motor. If the PMSM with a sinusoidal EMF is driven by a sinusoidal PWM inverter, the motor does not generate torque ripple but the effective utilization of the magnetic flux is poor. Although the PMSM with a trapezoidal EMF is improved in respect to the utilization of magnetic flux, the motor generates torque ripple even if the motor is driven by the sinusoidal PWM inverter. The proposed new modified trapezoidal modulating signal is able to decrease the torque ripple even if the motor has sinusoidal EMF or non-sinusoidal EMF. The new PWM strategy using the proposed modulating signal makes a great improvement in the characteristics in the three-phase PWM inverter for the PMSM drive. The characteristics and advantages of the new system are confirmed by theoretical analysis, simulation by modeling the system and experiments.

2. Harmonic Torque Analysis of PMSM

The torque τ of the PMSM is given by

$$\tau = \frac{P_m}{\omega_m} = \frac{e_u i_u + e_v i_v + e_w i_w}{\omega_m} \tag{1}$$

where, P_m is motor output power, ω_m is mechanical angular velocity of a rotor, e_u,e_v,e_w are EMF of u, v, and w phase, i_u,i_v,i_w are armature current of u, v, and w phase [5~7].

This paper describes the PMSM of a non-salient type whereby a d-axis inductance is equal to a q-axis inductance. The PMSM is a surface permanent magnet synchronous motor (SPM). If the armature current and the EMF of the PMSM are the same phase angle, the motor performs more efficient drives. In this condition, the motor output power P_m can be expressed by the following equation.

$$P_m = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} k_{mn} E_m I_n \tag{2}$$

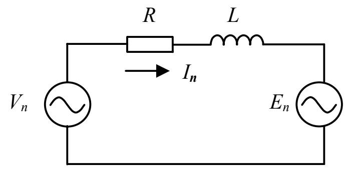


Fig. 1 Equivalent circuit of the PMSM (per phase)

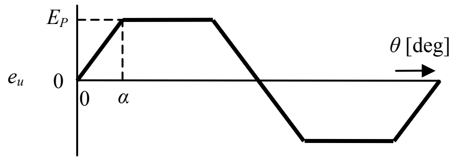


Fig. 2 Trapezoidal electro-motive force

$$k_{mn} = \cos(m-n)\omega_e t\left\{\frac{1}{2} + \cos\frac{2}{3}(m-n)\pi\right\} - \cos(m+n)\omega_e t\left\{\frac{1}{2} + \cos\frac{2}{3}(m+n)\pi\right\}$$

where, $m=6k\pm1(k=0,1,2,\ldots,m\neq-1)$, $n=6l\pm1(l=0,1,2,\ldots,n\neq-1)$, E_m is m-th harmonics component amplitude of EMF, I_n is n-th harmonics component amplitude of armature current and ω_e is electric angular velocity of a rotor. When the PMSM with the non-sinusoidal EMF is driven by the non-sinusoidal wave voltage, the output torque has harmonics including 6 times components of the inverter output frequency. If the torque due to the 6th harmonic torque could be eliminated, it would be possible to substantially reduce torque ripple. From (1) and (2), the 6th harmonic torque is obtained [7~9].

$$\tau_6 = \frac{3}{2}\frac{1}{\omega_m}\{E_1(I_7 - I_5) + (E_7 - E_5)I_1 + E_5 I_{11} + E_{11}I_5 + E_7 I_{13} + E_{13}I_7 + E_{11}I_{17} + E_{17}I_{11} + \cdots\} \tag{3}$$

Fig. 1 shows the equivalent circuit of the inverter-fed PMSM. Armature current I_n is calculated from the armature voltage V_n, the EMF E_n, and the motor impedance. I_n is given by

$$I_n = \frac{V_n e^{jn\delta} - E_n}{R + jn\omega_e L} \tag{4}$$

where, δ is the phase difference of the armature voltage V_n and the EMF E_n.

When the motor EMF is the trapezoidal waveform as shown in Fig. 2, the amplitude E_n of the n-th component of the EMF is

$$E_n = \frac{4E_P}{n^2 \pi \alpha} \sin(n\alpha) \tag{5}$$

where, n=1,3,5,…, E_P is the amplitude of trapezoidal wave and α is the angle of incline part of the trapezoidal wave.

The voltage equation of PMSM with a non-sinusoidal EMF wave is expressed by

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R+pL & -\omega_e L \\ \omega_e L & R+pL \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} e_d \\ e_q \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\omega_e t & \cos\left(\omega_e t - \frac{2}{3}\pi\right) & \cos\left(\omega_e t - \frac{4}{3}\pi\right) \\ -\sin\omega_e t & -\sin\left(\omega_e t - \frac{2}{3}\pi\right) & -\sin\left(\omega_e t - \frac{4}{3}\pi\right) \end{bmatrix} \begin{bmatrix} e_u \\ e_v \\ e_w \end{bmatrix}$$

where, v_d,v_q are d-axis and q-axis component of the armature voltage, i_d,i_q are d-axis and q-axis components of the armature current, e_d,e_q are d-axis and q-axis component of an EMF, R is the stator resistance and L is the stator inductance. Regardless of the position of a rotor, the L is constant [5,10]. If i_d is equal to 0, the armature current and EMF will take the same phase angle. Substituting i_d=0 into (6), ignoring a differential term, line-to-line voltage V_1' in a steady state is obtained by

$$V_1' = \sqrt{{v_d}^2 + {v_q}^2} \tag{7}$$

where v_d=-$\omega_e L i_q$, v_q=Ri_q+e_q. V_n of (4) is obtained by giving voltage so that the amplitude of the fundamental component of the armature voltage is equal to V_1' of (7). δ is given by

$$\delta = \tan^{-1} \frac{\omega_e L i_q}{R i_q + e_q} \tag{8}$$

The 6th harmonic torque τ_6 is calculated by substituting the motor parameters into the above equation.

3. Characteristics of the PMSM

3.1 A Conventional PWM Strategy

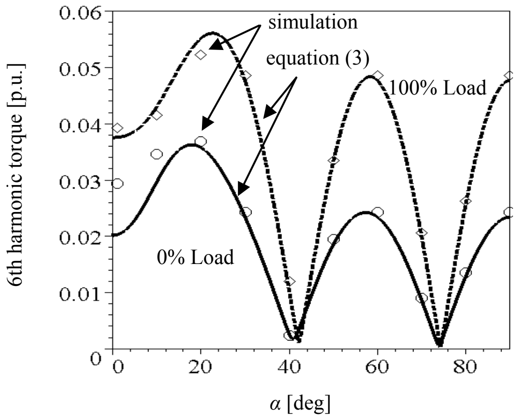


Fig. 3 Amplitude of 6th harmonic torque as a function of α.

Table 1 Specifications of PMSM

Rated power	1.5kW
EMF at rated speed	115Vrms
Number of pole pairs	3
Rated frequency	100Hz
Rated current	7.5A
Stator inductance	4.8mH
Stator resistance	0.775Ω
Moment of inertia	0.015kgm^2
Rated torque	7.16Nm

In the sinusoidal PWM inverter fed PMSM, the motor with sinusoidal EMF generates constant torque. To obtain the sinusoidal EMF of the PMSM, the motor is adapted to the distributed winding, the skew slot and the design of the shape of a permanent magnet. Therefore, PMSMs widely use non-sinusoidal EMFs because sinusoidal EMFs cause an increase in cost and a decrease in EMF.

Typical parameters of low power PMSMs are listed in Table 1. Fig. 3 shows the 6th harmonic torque obtained by using the parameters of Table 1. A supply voltage is a sinusoidal wave and the output frequency is 5Hz. The circle and diamond marks in Fig. 3 show the simulation results. A simulation block diagram is shown in Fig. 4. To obtain the armature voltage by (7), the d-axis component of the armature current is kept at i_d=0. The motor output power P_m divided by the EMF e_q gives the torque current i_q.

Simulation results concurred with the calculated line. From Fig. 3, it is found that the PMSM with trapezoidalEMF of α=42 degree can reduce the 6th harmonic torque. Additionally, point α increases torque ripple even if the motor is driven by the sinusoidal PWM inverter.

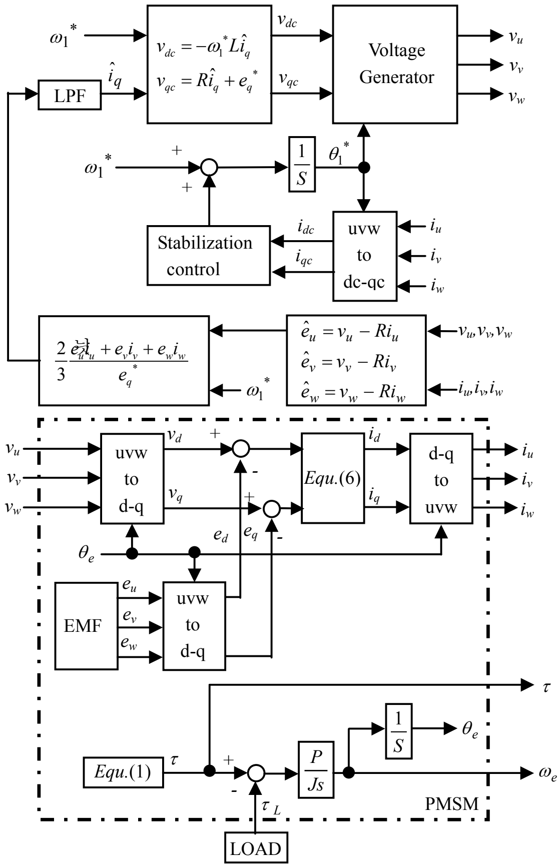


Fig. 4 Simulation block diagram of the motor-torque ripple

3.2 Proposed PWM Strategy

3.2.1 Modified trapezoidal modulating signal

Fig. 5 shows the generation principle of the PWM pulses for the power inverter to reduce the torque ripple of the PMSM. In Fig. 5(a), a modified trapezoidal modulating signal is obtained by superposing a rectangular wave on the specific trapezoidal wave whose flat portion is 120 degrees. γ is a ratio of the trapezoidal waveform to the rectangular waveform superposed on the trapezoidal waveform. A modified trapezoidal waveform becomes a rectangular waveform when γ= 1, and the waveform is a trapezoidal wave when γ = 0.

Fig. 5(e) shows the PWM output waveform that includes the fundamental component, higher-order harmonics and sidebands of the carrier frequency [1,3]. Amplitude of the fundamental component and the higher-order harmonics can be obtained by analyzing the waveform, as follows;

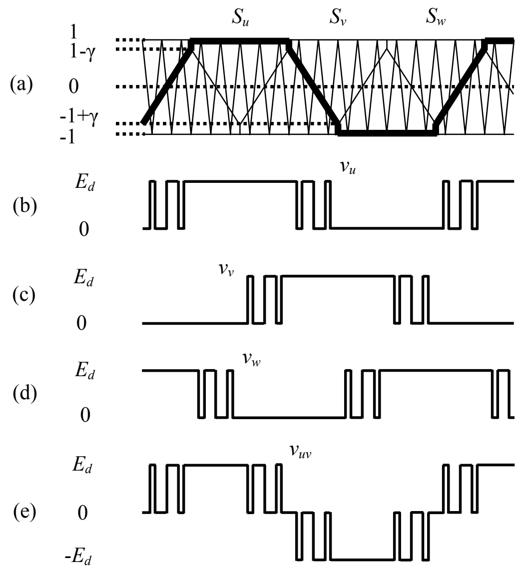


Fig. 5 Generation principle of PWM pulses

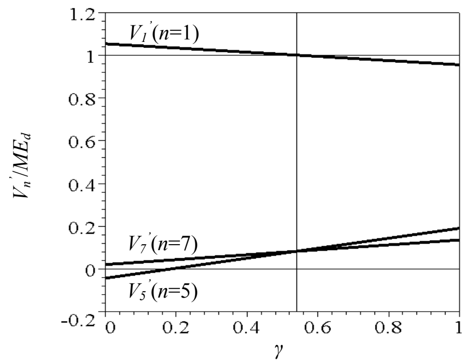


Fig. 6 Major frequency components included in the output PWM voltage

$$V_n' = ME_d\left\{\frac{4\gamma}{n\pi}\cos\frac{n}{6}\pi + \frac{24(1-\gamma)}{n^2\pi^2}\sin\frac{n}{6}\pi\right\}\cos\frac{n}{6}\pi \qquad (9)$$

where, $n=6k\pm1$(k=0,1,2,…,n≠-1), E_d is a DC link voltage. As seen in (9), the amplitude value V_n' of the fundamental component and the harmonic content of the PWM output voltage is the function of γ. Fig. 6 shows major frequency components included in the output PWM voltage as a function of γ.

3.2.2 Characteristics of the PMSM with a sinusoidal EMF

In the PWM inverter using a modified trapezoidal modulating signal fed PMSM, the motor with sinusoidal EMF is investigated. Fig. 7 shows the relation between γ and the 6th harmonic torque. Output frequency is 5Hz. As seen in Fig. 7, minimum amplitude of 6th harmonic torque occurs at γ = 0.42. The fundamental component of the output voltage for the modified modulating signal of γ = 0.42 increased about 17% more than that of the PWM strategy by a sinusoidal modulating signal.

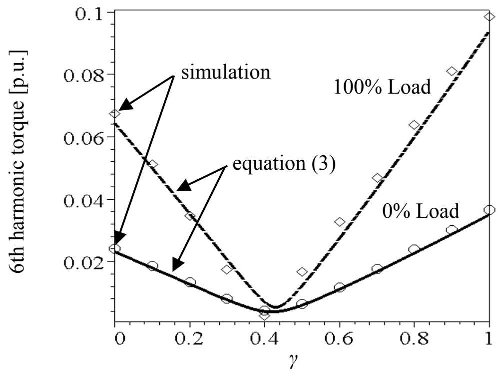


Fig. 7 Amplitude of 6th harmonic torque as a function of γ

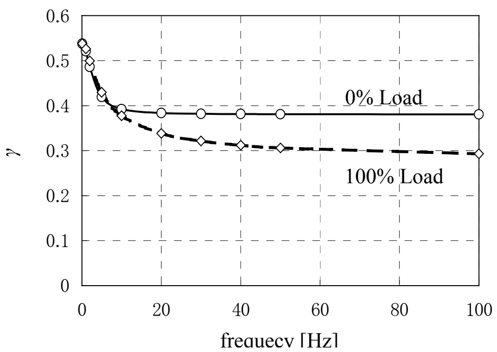
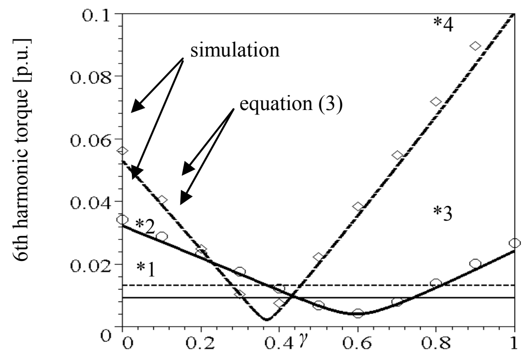


Fig. 8 γ of minimum 6th harmonic torque

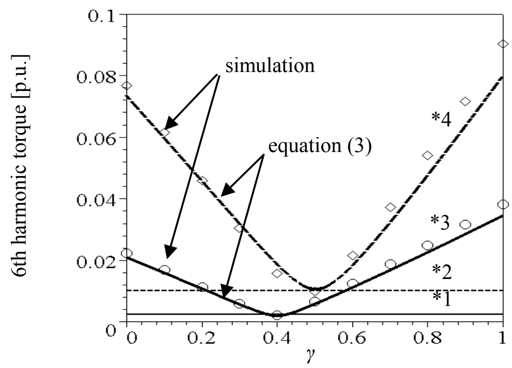
Fig. 8 shows the γ values which occur at the minimum 6th harmonic torque for various output frequencies of the inverter. The γ values, which yield minimum 6th harmonic torque, are decided by the output frequency and load conditions. Since the pattern of optimal γ can be calculated in advance, it is easy to control it by using the table of software.

3.2.3 Characteristics of the trapezoidal EMF

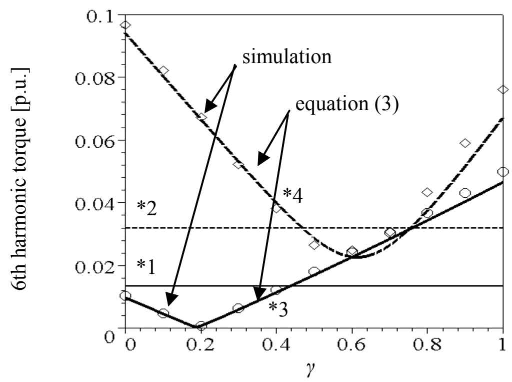
Fig. 9 shows the relation between γ and the 6th harmonic torque in three EMF cases. The α values of the trapezoidal EMF are 45degrees, 40degrees and 35degrees. The output frequency of the inverter is 5Hz.



(a) Trapezoidal EMF, α=45degrees



(b) Trapezoidal EMF, α=40 degrees



(c) Trapezoidal EMF, α=35 degrees

Fig. 9 6th harmonic torque

*1 Sinusoidal power supply (0% load), *2 Sinusoidal power supply (100% load), *3 Modified trapezoidal (0% load), *4 Modified trapezoidal (100% load)

When the PMSM with trapezoidal EMF is driven by sinusoidal voltage, the 6th harmonic torque is generallylarger than the modified trapezoidal voltages for γ occurring minimum values. Therefore, if the PMSM with trapezoidal EMF are driven by the modified trapezoidal

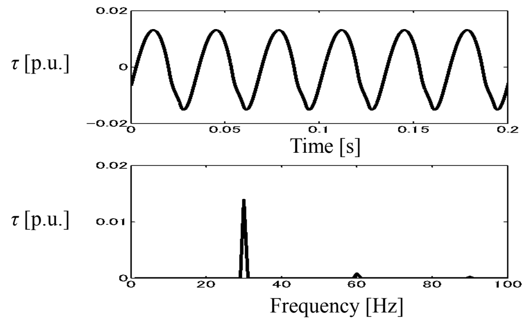


Fig. 10 Simulation results of the sinusoidal voltage

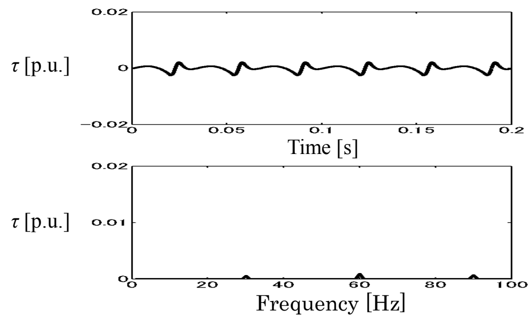


Fig. 11 Simulation results of the proposed voltage (γ=0.19)

voltage, the harmonic torque can be decreased by changing the γ according to α of a trapezoidal wave.

Fig. 10 shows a simulated torque waveform and its frequency spectrum when the PMSM with α = 35degrees trapezoidal EMF is driven by a sinusoidal voltage, and Fig. 11 is a waveform and spectrum when the PMSM with α = 35degrees trapezoidal EMF is driven by a modified trapezoidal voltage of γ=0.19. The amplitude of the 6th harmonic torque for sinusoidal voltage is 1.4%, and for modified trapezoidal voltage is only 0.1%.

4. Experimental Results

The principal parameters of the tested PMSM are listed on Table 1.

Fig. 12 shows the EMF waveform (line-to-line voltage) of the PMSM. Where a rated speed is 2000r/min and line-to-line EMF voltage is 115V. The amplitude of the 5th harmonic voltage of the EMF is 2.4 % of the fundamental component, and the 7th harmonic is 0.8%.

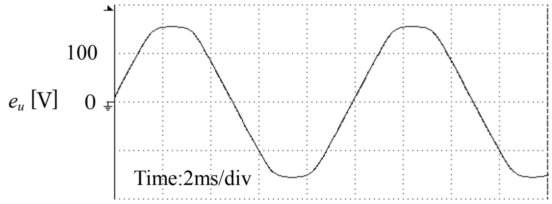


Fig. 12 EMF waveform (line-to-line)

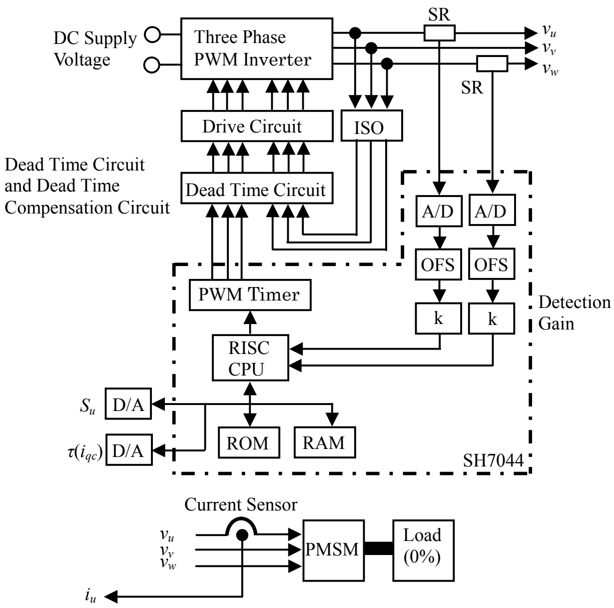


Fig. 13 Experimental Circuit
SR=Shunt Resistance and Isolation Amplifier
ISO= Isolation Circuit

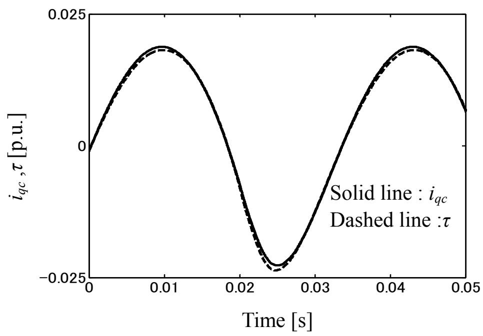


Fig. 14 Comparison of i_{qc} and torque

Fig. 13 shows a system configuration for the experiments. "Dead Time Circuit" is shown in Fig.13. The dead time influence of the inverter is very small because of a voltage feedback system. The carrier frequency of the PWM inverter is 2kHz and the motor condition is no-load. The OFS is the offset correction of the detected current. In general, it is difficult to measure the harmonic torque directly. Fig. 14 shows the simulation wave of the torque current i_{qc} and the torque. The voltage of the PMSM is a sinusoidal wave and the EMF is a trapezoidal wave with α = 32degrees. The output frequency is 5Hz and the load is 0%. The torque agrees well with the torque current i_{qc}. Therefore, in this experiment, the harmonic torque is measured by using torque current i_{qc}. In Fig. 13 , S_u is a modulating signal, $\tau(i_{qc})$ is the qc-axis current of a control-coordinates axis. And it is substituted for the torque.

Fig. 15 shows the experimental waveforms for a sinusoidal modulating signal, and Fig. 16 is the experimental waveforms of a modified trapezoidal modulating signal (γ=0.4). In Fig. 15 and Fig. 16, (d) is the frequency spectrum of $\tau(i_{qc})$.

For the PWM strategy using the sinusoidal modulating signal of Fig. 15, the 6th harmonic torque has appeared clearly in a torque waveform and its frequency spectrum. The harmonic torque is reduced in the PWM strategy by the modified trapezoidal modulating signal of Fig. 16.

Fig. 17 shows the amplitude of the 6th harmonic torque for various γ of a modified trapezoidal signal. For the modified trapezoidal of γ=0.4, the 6th harmonic torque is reduced about 1% compared with the sinusoidal modulating signal. The experimental results are in agreement with the analysis output Fig. 9 (b) in case α is 40 degrees.

As a result, the modified trapezoidal modulating signal is suitable for the PMSM with a sinusoidal EMF. In the PMSM with a non-sinusoidal EMF, the proposed method is still more effective.

5. Conclusions

A new modified trapezoidal modulating signal suitable for the PMSM drive has been proposed. The proposed new modified trapezoidal modulating signal is able to decrease torque ripple even if the motor has sinusoidal EMF or non-sinusoidal EMF. By using the proposed modulating signal, the system reduced torque ripple as well as achieved the effective utilization of the DC supply voltage for the inverter. Many improvements are accomplished by the PWM strategy which adapts the modified trapezoidal modulating signal without a change in hardware.

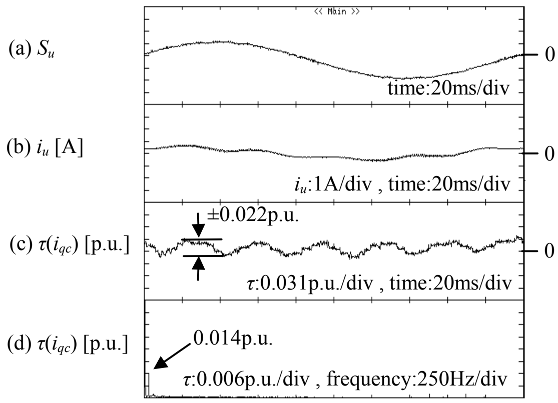


Fig. 15 Experimental Waveforms (Sin modulating signal)

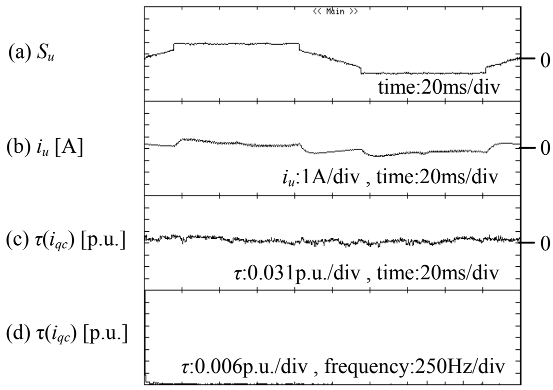


Fig. 16 Experimental Waveforms
(Modified trapezoidal modulating signal, γ=0.4)

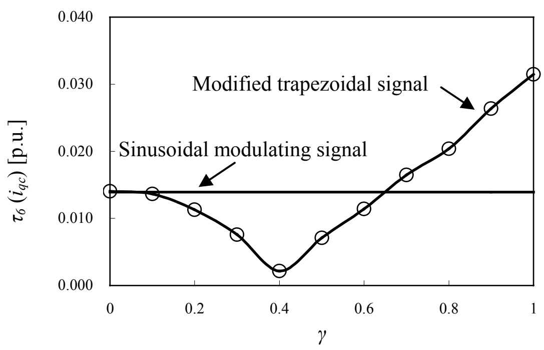


Fig. 17 Relation between γ and 6th harmonic torque
(Experimental value)

The characteristics and advantages of the new system have been confirmed by theoretical analysis, simulation by modeling the system and experiments for the drive system of PMSM.

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